## Teaching Math

ONUS WEB CONTENT to accompany
The ESL/ELL Teacher's Survival Guide, Second Edition by Larry Ferlazzo and Katie Hull Sypnieski © 2022 John Wiley \& Sons

> NOTE: This chapter appeared in the first edition of the ELL/ESL Teacher's Survival Guide. Although we replaced it with an updated chapter written by another math teacher, we still believe that this is an excellent chapter that readers would find useful.

A mother crab noticed several different animals walking straight one day. She then saw her son moving sideways towards her. The mother decided that he should start walking straight. The little crab tried repeatedly, but failed. He asked his impatient mother to model walking straight for him. She noticed that she made no progress trying to walk straight, but knew she would move quickly walking sideways.
"I give up," she finally admitted. "I just don't think nature intended us all to walk the same way." ${ }^{1}$

This story illustrates that each of us has a unique way of getting from one place to another. Each student brings unique talents from his or her individual strengths and from his or her cultural background. This principle is true for all students but is particularly valuable when teaching English learners, who come from all over the world. The fund of knowledge that these students bring from their unique experiences can become the basis of a strong math program for English learners.

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Our colleague, Lorie Hammond, shares her ideas in this chapter on how to tap this fund of knowledge. This chapter was written by Lorie Hammond, professor of teacher education at CSU Sacramento and the cofounder and academic director of Peregrine School, a project-based bilingual school for preschool through eighth grade in Davis, California (see PeregrineSchool.org).

## MAKING MATH RELEVANT

Newcomer students in a middle-school English language learners math class pored over the want ads from our local newspaper, attempting to figure out how they would spend their monthly "income," were they to live on their own. Each student had drawn a different job, with its corresponding salary, from a random selection. Their task then became how to meet their monthly needs for housing, food, transportation, and recreation on the salary they received.

Many discussions ensued. Some students from Thailand were surprised that they would have to spend their money on housing and food, since in their country these were things their family produced for themselves. "What did you need money for, then?" a Russian student asked. To our surprise, the Thai student responded that money was for buying a new wife or child. This immediately triggered discussion among all the students about whether women and children should be bought and sold and about how differently money is used here compared to some other countries.

Russian Christian students from very large families (10 to 12 children on average) planned to rent apartments by themselves. They loved the idea of their own individual space. For other students, from more collective cultures, this was a strange and scary idea. They chose to pool their incomes and live together in a big house.

What if the salary a person received was not enough to rent an apartment and buy a car? What kind of education was needed to get jobs that supplied the income needed for a good lifestyle? As newcomers to the United States, having arrived in the last three months, students became engaged in thinking about planning for their futures and began to connect education in this country to obtaining a good job.

Through this exercise, the teacher introduced students to an approach to mathematics that was relevant to their lives, that involved divergent solutions, that encouraged dialogue, and that engaged students as active learners. In this chapter, this example and others will be used to analyze how math might best be taught to English language learners and mainstream students in secondary schools. The five steps of the Organizing Cycle will again be used as a guide.

## BUILDING RELATIONSHIPS WITH STUDENTS AND ACCESSING PRIOR KNOWLEDGE

Mathematics is a gatekeeper in many secondary schools. Math proficiency determines not only which math class students will be assigned to, but also whether they can gain access to laboratory science classes and to an academic curriculum in general. For English language learners, the situation is even more complex.

Students' language levels do not necessarily correspond to their levels of proficiency in mathematics. Factors such as whether students come from a developing country, a situation of poverty or war, and whether their experience is rural or urban are often key to understanding their prior knowledge in mathematics. It is particularly important to get to know one's ELL students because they may be much more competent in mathematics than they at first appear, since they will likely have trouble deciphering word problems and some symbolic codes in English, causing them to score low on standard tests.

On the other hand, some students from rural areas or poverty, or both, may have attended school for a short time or not at all and may never have learned mathematical conventions common in elementary schools worldwide. To add to the complexity, some students use symbols and algorithms differently because of the conventions in their countries. (In some countries, for example, people use commas where we use periods and vice versa-for example, $1,000.00$ would become $1.000,00$.) These students will need to be taught to represent numbers in ways that will be understood in the United States.

Students' math skills can also be strongly affected by their competency in English. There is both truth and falsehood in the statement that mathematics is a "universal language." On the one hand, mathematics is its own language, which all students need to learn from scratch in school. Some parts of this language can be manipulated using numbers alone. Thus, when English language learners are mainstreamed into regular classes with English-speaking peers, the first subject they can adjust to is mathematics, since students who are well trained in working with numbers can transfer their skills to the English-speaking context. On the other hand, students at each competency level in English may display different strengths and weaknesses in mathematics. For example, students new to English will do better when yes-or-no responses and responses in their own language are allowed, whereas advanced English learners will be able to participate in conversations about math problems.

All of these factors make clear that teachers must learn about the factors from their students' past experiences, as well as assess their current academic levels, in determining the math placements appropriate to their needs. Activities such as the want ads project described earlier help ELL students, as well as non-ELLs, to do

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math without severe anxiety and solve problems in their own way. This in turn allows teachers to analyze (1) students' ability to express ideas about mathematics in English or in some other modality, such as through a graph or diagram and (2) students' background knowledge in formal and informal mathematics.

Learning about students' prior experiences with math can be helpful in determining the instructional strategies to best meet individual student needs. It is not uncommon for teachers of ELLs to experience two extremes in student prior experience, with many students falling in between.

The first extreme is the student who comes from a country that is very strong in mathematics, perhaps stronger than the United States. (Students from Central Asia, East Asia, and the former Soviet Union, for example, often fit this description.) In this case, the central decisions schools must make are equity issues: making sure that English language learners, despite their language ability, can access advanced mathematics classes at their high school or middle school and will receive teaching there that they can comprehend.

The other extreme in student prior experience is the learner who comes from a rural area of a developing country, from a hill tribe or other fourth-world people, or from a war-torn country, and who has had little or no formal schooling before entering school in the United States. This student will not know the basic skills in mathematics typically learned in elementary school though she may know other ways to solve mathematical problems-ways that are used in her group. This situation, along with a less extreme but common situation of a student who has received a partial mathematics education, can be addressed by the strategies listed next, as we make our way through the Organizing Cycle.

An exciting way to begin is to use the tools of a small but growing field called ethnomathematics (Favilli (n.d.); International Study Group (n.d.)). This field is a blend of cultural anthropology and mathematics and involves teachers in interviewing students, and students in interviewing family members, about cultural means of solving mathematics problems, followed by the sharing of this information as part of the mathematics classroom curriculum. What is found can be as simple as different conventions of notation or different algorithms, such as other ways of doing long division. In some cases, however, the approach can shed light on deep cultural differences.

In any case, an ethnomathematical approach, in which teachers explore what students and their families already know about mathematics, rather than assuming that they begin with a deficit in relation to conventional American skills, can improve relationships between teachers, parents, and students dramatically, because it shows such respect for and interest in students' cultural knowledge. It can also be very interesting for everyone involved. And again, it can also be an engaging strategy to use with mainstream students who often share a classroom with ELL students.

## IDENTIFYING AND MENTORING STUDENTS' LEADERSHIP POTENTIAL AND LEARNING BY DOING

In planning a mathematics curriculum for English language learners, it is important to understand that mathematics is a conceptual subject that can only be understood when students discover answers for themselves. While algorithms play an important role in making mathematics efficient, the real study of mathematics is not the memorization and practice of algorithms but the understanding of why the algorithms work.

While teachers should give mini-lessons that explain the work that students will do and that target misconceptions or mistakes teachers observe, much of the time in the mathematics class can be spent in active exploration by students individually or, more often, in small groups.

Allowing students to work in groups to solve mathematical problems empowers students and allows them to exert leadership. English language learners need to practice their communication skills, regardless of the subject learned, rather than to work quietly. In fact, language is learned only through communication. Hence, group work enables students to be leaders, organizing how to solve problems, and forwards their general goal of learning English. In some cases, when concepts are hard to understand, students might be grouped with others who speak their own language, so that their discussion of concepts can be more fluid and deep. Students can then translate what they have discovered back into English.

A lesson cycle for a 50 -minute class might go as follows:

- Teacher introducing the problem and/or concept (five minutes)
- Students working in groups to solve a problem or problems, recording their work and solutions on a large piece of paper ( 30 minutes)
- Student groups presenting their results to other groups, followed by a reflective discussion among the groups ( 15 minutes)

In this style of constructivist teaching, in which students construct their own solutions, problems should be sufficiently challenging and should have various possible solutions. Manipulatives of various sorts should be available to help students solve their problems. These might include counting cubes, place value cubes, geometric figures, rulers, compasses, triangles, and even common objects, such as beans, that can be arranged and counted.

Students must receive scaffolding that helps them to organize their problem solving. Since mathematics involves three languages-visual-geometric, equations, and words-a repeated worksheet such as the one offered here can help students to use all three in the solution of every problem (Exhibit 10.1).

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## EXHIBIT 10.1. Mathematics Worksheet

Name $\qquad$
Date $\qquad$
Problem: (already written)
Drawing of how you solved your problem (can be a sequence of drawings):
Equation that explains your problem and solution:
Description of what you did in words:

Students new to English will require support in filling out this type of worksheet, which can be completed individually or as a group, depending on teacher goals. This help can come in several forms: a word wall with key terms that students will need to use as a reference, help from other students, allowing phrases rather than sentences or labeled cartoons for students with very little English, or assistance from the teacher. This kind of writing benefits ELLs and non-ELLs alike. Research has found that incorporating writing into mathematics instruction "raises the 'cognitive bar,' challenging students to problem solve and think critically" (Urquhart, 2009, p. 4).

Student leadership can also be developed by assigning changing roles during group times, so that students can take turns organizing their group, being the scribe, or taking responsibility for other tasks.

## REFLECTION

In some countries, mathematics curricula are described as going very deep. Only a few concepts are presented each year, but they are explored in depth. In contrast, US mathematics textbooks tend to cover many concepts in quick succession, creating an effect that is "an inch deep and a mile wide." This is important for all students, but especially for English language learners who have missed earlier years of schooling and need to catch up in high school. These students will have a less daunting task if teachers can isolate the key concepts that need to be learned, rather than expecting them to wade through extensive materials.

A program in which students solve problems and then pause to share their solutions with each other and reflect upon their new understandings can be powerful in
helping students to understand mathematics deeply and therefore gain mastery quickly. Reflective discussions among students are therefore an essential part of good mathematics teaching. The reflective activities recommended in Chapter Four can also be used in the mathematics classroom to promote this kind of thinking and discussion.

Since students' mathematics abilities are not limited by their English levels, it falls to the teacher to figure out how to communicate across language barriers so that students can reach their full mathematics potential. Data shows that English language learners initially test below their peers in mathematics, but once fluent these students are able to accurately demonstrate their higher math skills (Kopriva, 2003). Therefore, finding effective ways of teaching mathematics to all students, regardless of English level, is a true equity concern.

Additional resources, including links to interactive online math exercises, can be found on our book's web site at www.josseybass.com/go/eslsurvivalguide.

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## Note

1. Adapted from "The Crab and Its Mother," an Aesop fable. Retrieved from http:// storywise.com.sg/storytelling/story-the-crab-and-its-mother
